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Research article

Mathematical Analysis of Modeled Direct Real Estate Investment

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Abstract

The Asset Dividing Appraisal Model (ADAM) enables the appraisal of cash flows resulting from direct real estate investments. The model is an evaluation tool, which takes capital markets and the specific characteristics of real estate as an asset into consideration, while also considering different ownership approaches of real estate in the European Union. Thus, it contributes to the harmonization of capital markets and of direct real estate investment evaluation as intended by the "European Directive on Markets in Financial Instruments 2004/39/EC". ADAM is based on financial mathematical instruments and on the property valuation methods of different cultural areas. It combines continental European (income method) and international (DCF) property valuation approaches. A mathematical analysis based on empirical data confirmed the validity of the methodology of the model. In the course of the analysis the major input variables that determine the results of the model and how the model reacts to marginal deviations of input data, were quantified. This was done using partial derivations and a simulation study.

Keywords: Direct Real Estate, Investment Appraisal, Mathematical Analysis, Partial Derivations, Simulation-Study

Introduction

The Asset Dividing Appraisal Model (ADAM) (Schäfer, 2013) is designed to evaluate direct real estate investments, while considering land and building separately. Thus, it provides a method for complex analyses of direct property investments which takes the capital market oriented view of property as an asset, as well as the special characteristics of property (such as site-specificity, heterogeneity, uniqueness, limited substitutability, interdependence and long production period etc.) as an asset into account. Considering the background of the different property ownership laws in the EU (for example in the UK or Czech Republic, where ownership of land and building can theoretically and factually differ and on the contrary to Germany where ownership of land and building is consolidated by law) (Adam, Rombach, 2004 ; Kopp, Waldner, 2004), this model can contribute to the harmonization of European property and capital markets and make them more transparent which is the intention of the "EU Directive 2004/39/EG on Markets in Financial Instruments". Furthermore it could be applied to the upcoming consolidation of property ownership in the Czech Republic in 2014, which will unite legal ownership of land and building of real estate (Hrncir, 2012). The purpose of this paper is to evaluate and analyze ADAM by considering empirical data. The aim of the performed mathematical analyses is to find out how the model reacts to marginal deviations of input data. Additionally, it is the goal to determine which input variables actually are the major influence factors of the calculated Asset Rate of Return (ARR) of the model

Materials and Method

Within the context of ADAM (Schäfer, 2013) real estate is treated as a fixed asset, which consists of land and the buildings, located on the land. Thus, the economic lifetime of the land is separated from the economic lifetime of the buildings and is considered to be indefinite (Metzger, 2010 ; Kleiber, 2010). The economic lifetime therefore deviates from the remaining useful life as defined in the income capitalization approach in the German law §17 para.1 (2) ImmoWertV. The economic use of a property, on which the economic lifetime is based, describes a type of use, which generates cash flow (rental income). Here, the economic use itself is based on provisions in tenancy agreements and relates to both the land (ground rent, perpetuity) and the use rent (resulting from the building) of which a property consists. A separate valuation of land and buildings is conducted according to the German income capitalization approach (Moll-Amrein, 2009; Metzger, 2010; Kleiber, 2010), while the international DCF method considers land and buildings as one (Naubereit, 2009) or, to express it another way, DCF pays no special attention to the ground rent. The German income capitalization approach identifies the market value of a property in accordance with ImmoWertV. It thus rescinds the imperfection of the property markets by employing a methodology, which takes the segmentation of the property markets (local sub-markets) into account by using parameters like the specific local property yield. Here, the economic lifetime is considered to be between 60 and 80 years and is therefore limited. The value of the land is seen as indefinite and as perpetuity (Metzger, 2010; Kleiber, 2010; Moll-Amrein, 2009). The increasing interconnection of property assets and capital markets is not taken into consideration by the income capitalization value method. The DCF method evaluates a property as an asset seen from a capital market oriented perspective and can produce various different results according to the selection of the discount rate. It is, for example, possible to discount with a "risk-free capital market interest" with the addition of a risk premium. As a rule this appraisal of the property as an asset from an international capital market perspective assumes a property lifetime of 10 years (depending on the rental agreement) (Kleiber, 2010). When discounting with an opportunity interest rate, the DCF approach can also determine a subjective value, which is decisive for the approval or rejection of a property investment (Engel, 2002 and 2003). Regarding the DCF approach Kleiber (2004) states the following: "Nevertheless the Discounted Cash flow Approach and its areas of application have certain advantages. The approach originates from business investment analysis and has always been employed in Germany, in particular, for the assessment of basic business figures, which deal with the determination of business targets. It is exactly in this sense that international literature identifies those cases in which it is a question of a subjective value for a particular investment as cases in which it is to be applied". The international DCF approach, with its origins in neoclassical finance and business evaluation (Spreemann, 2010), does not consider the specific characteristics of property as a business asset, or the perpetuity of the land value (Kleiber, 2010). The extremely varied and complex characteristics of properties (site-specificity, heterogeneity, uniqueness, limited substitutability, interdependence and long production period etc.) illustrate that we are here confronted with profoundly imperfect, very complicated and highly segmented markets, which, in addition, are neither organized, nor standardized (Moll-Amrein, 2009). Due to this fact, it is essential to conduct an investment appraisal, which takes the individual market segments into account. However, growing globalization, internationalization and capital market orientation, or rather, the merging of capital markets with property markets (Beyerle, 2007) also calls for an approach, which incorporates internationally comparable assets. Property is increasingly becoming an object of capital market-based investment products such as real estate PLCs and investment trusts (Reits), real estate private equity (REPE), special funds etc. (Gantenbein, 2011). These two (in principle contrary) effects provide the necessity of the model presented here. The valuation of a real estate investment, meeting the requirements of the capital markets and special characteristics of the real estate asset, is thus made possible.

The author describes the model methodologically and inductively as follows:

$$Z_0 = \sum_{t=1}^{T} \frac{E_t - A_t}{(1+p)^t}, \quad A_0 = B + G, \quad r_E = \frac{\left[(1+p)^T - 1\right] \cdot z \cdot B + Z_0 \cdot p \cdot (1+p)^T}{\left[(1+p)^T - 1\right] \cdot A_0}, \quad R_E = r_E \cdot A_0$$

Formula 1: Asset Dividing Appraisal Model, ADAM (Schäfer, 2013)

Z = cash-capital p = calculation rate

E_t = cash inflow in t A_t = cash outflow in t

A_0 = acquisition costs B = property (ground)

G = property (building) r_E = Asset Rate of Return (ARR)

T = economics lifetime z = yield of property (ground)

R_E = Asset Rent Generation (ARG)

At first the model is presented in a three-dimensional space depending on 2 different input variables respectively. Afterwards the partial derivation of the independent variables are formed and visualized above the definition area of the respective variables. Finally a simulation study is conducted based on the empirical values of the input factors, which simulates these factors. Based on the simulated values empirical correlations between the simulated data of the input variables and the therefrom resulting Asset Rate of Return *ARR* can be calculated in order to evaluate the intensity and direction of the connections. For this reason domains were derived for the 5 independent variables. For this different source such as the IPD Annual Lease Review 2012, the real estate report of expert committees in Hessen and the interest rates for borrowed capital of different real estate financiers were consulted. The main focus of this paper is the mathematical analysis of the model. Hence the discussion regarding the detailed derivation of the domains is relinquished in this paper. The domains of the independent variables present themselves as follows:

$D_B = \{B \in \mathbb{R} \mid 10.000 \geq B \leq 450.000\}$

$D_G = \{G \in \mathbb{R} \mid 38.000 \geq G \leq 2.272.000\}$

$D_z = \{z \in \mathbb{R} \mid 0{,}0111 \geq z \leq 0{,}0276\}$

$D_p = \{p \in \mathbb{R} \mid 0{,}0252 \geq z \leq 0{,}0856\}$

$D_T = \{T \in \mathbb{R} \mid 1 \geq T \leq 20\}$

As an additional base for the mathematical analysis a model calculation with the following values for the input variable is used:

$p = 0.065$ $z = 0.025$ $B = 120.667$ $G = 195.213$ $T = 14$

The cash-capital Z is a result of the following payments:

t	E_t [€]	A_t [€]	*t*	E_t [€]	A_t [€]
1	37.481	17.333	8	41.521	25.297
2	37.695	19.224	9	42.229	25.530
3	38.095	20.308	10	43.760	25.291
4	38.103	21.328	11	45.572	24.925
5	38.351	22.403	12	48.995	24.949
6	39.627	23.268	13	53.205	25.054
7	40.695	24.173	14	55.312	24.937

Table 1: Cash-Ins and Cash-Outs

Results and Discussion

In this section the mathematical analysis will be conducted as described previously.

Depiction depending on 2 different input variables in a three-dimensional space.

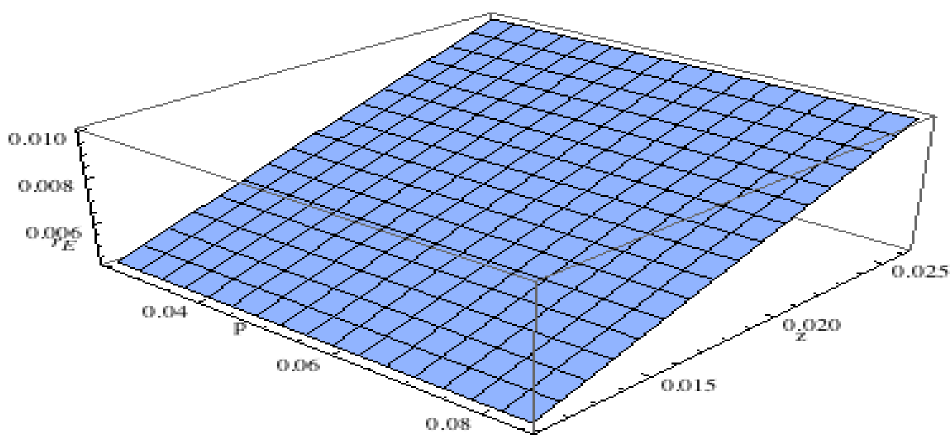

Image 1: *ARR* depending on *z* and *p*. $T = 14$; $B = 120667$; $G = 195213$.

Depending on *p* and *z* a linear effect on the Asset Rate of Return (ARR) over the domains of *p* and *z* can be seen.

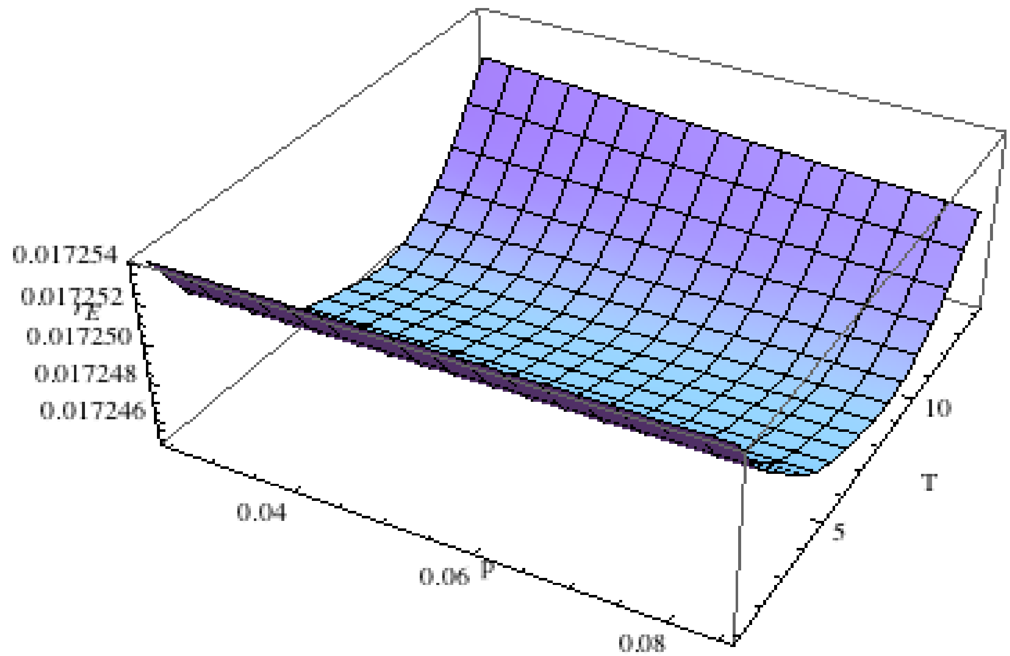

Image 2: *ARR* depending on *p* and *T*. $z = 0.025$; $B = 120667$; $G = 195213$.

Depending on *p* and *T* a strong non-linear effect on the ARR can be seen. For the constant value of *p* the ARR decreases in a non-linear fashion until $T = 8$. It then increases again in a non-linear fashion.

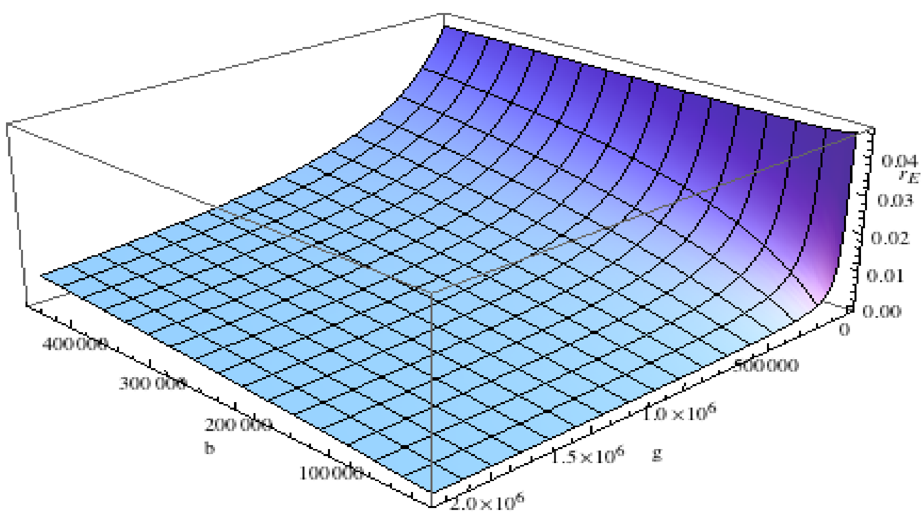

Image 3: *ARR* depending on *G* and *B*. $z = 0.025$; $B = 120667$; $G = 195213$

When depending on B and G a non-linear effect on the ARR can also be seen.

Forming the partial derivation and visualization of the marginal deviation.

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The first degree of the partial derivation was formed for every input factor to analyze the effect of the input factors on the ARR. This derivation reflects the so-called marginal effect an independent variable has on a dependent variable.

The marginal effect is the derivation of the dependent variable, which occurs when the independent value changes, or deviates by exactly one unit. To be sure that the observed effect is implied by only one independent variable all of the other respective input variables were fixed to the model calculation as follows:

$p = 0.065$ $z = 0.025$ $B = 120.667$ $G = 195.213$ $T = 14$

$D_B = \{B \in \mathbb{R} \mid 10.000 \geq B \leq 450.000\}$

$D_G = \{G \in \mathbb{R} \mid 38.000 \geq G \leq 2.272.000\}$

$D_z = \{z \in \mathbb{R} \mid 0{,}0111 \geq z \leq 0{,}0276\}$

$D_p = \{p \in \mathbb{R} \mid 0{,}0252 \geq z \leq 0{,}0856\}$

$D_T = \{T \in \mathbb{R} \mid 1 \geq T \leq 20\}$

Partial derivation of ARR according to B

$$\frac{z}{b+g} - \frac{b\left(-1+(1+p)^T\right)z + p(1+p)^T \sum_{t=1}^{T} (1+p)^{-t}\left(-a_t + e_t\right)}{(b+g)^2\left(-1+(1+p)^T\right)}$$

Formula 2: Partial derivation *ARR* according to *B*

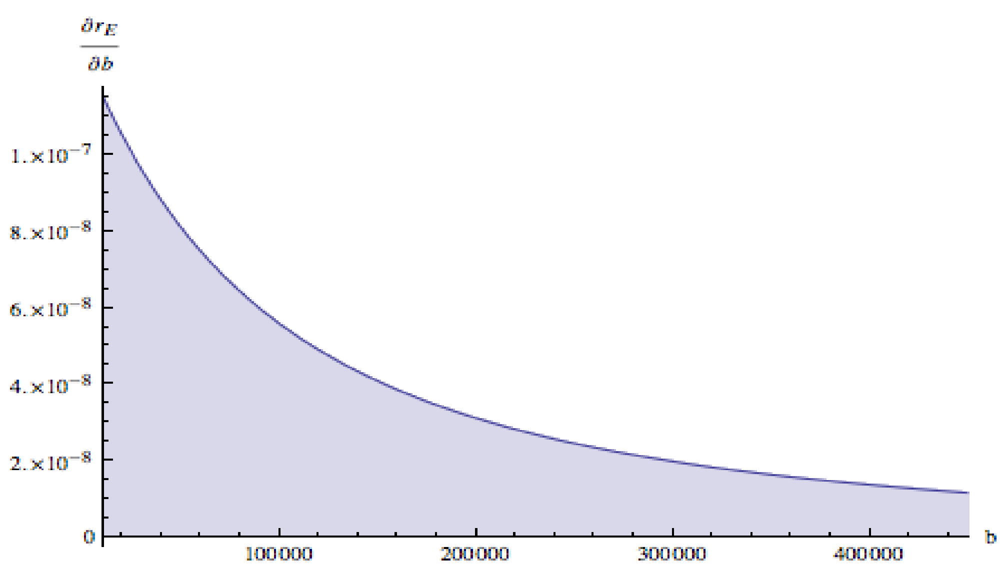

Image 4: Partial derivation of *ARR* according to *B*

As can be seen above the marginal effect of *B* on *ARR* increases with a decreasing *B.* The y-axis depicts the influence parameter on the independent variable.

Partial derivation of ARR according to G

$$\frac{b\left(-1+(1+p)^{T}\right)z+p(1+p)^{T}\sum_{t=1}^{T}(1+p)^{-t}\left(-a_{t}+e_{t}\right)}{(b+g)^{2}\left(-1+(1+p)^{T}\right)}$$

Formula 3: Partial derivation of *ARR* according to *G*

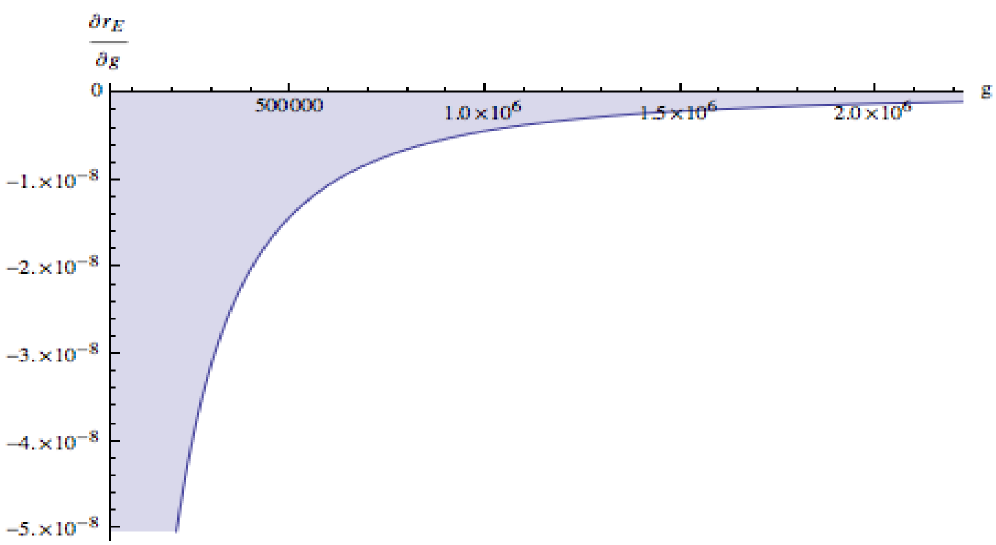

Image 5: Partial derivation of *ARR* according to *G*

The marginal effect of *G* on *ARR* is negative according to the visualization of the partial derivation of the first the degree

Partial derivation of ARR according to p

$$-\frac{(1+p)^{-1+T}T\left(b\left(-1+(1+p)^{T}\right)z+p(1+p)^{T}\sum_{t=1}^{T}(1+p)^{-t}\left(-a_{t}+e_{t}\right)\right)}{(b+g)\left(-1+(1+p)^{T}\right)^{2}}+$$
$$\frac{b(1+p)^{-1+T}Tz+(1+p)^{T}\sum_{t=1}^{T}(1+p)^{-t}\left(-a_{t}+e_{t}\right)+p(1+p)^{-1+T}T\sum_{t=1}^{T}(1+p)^{-t}\left(-a_{t}+e_{t}\right)+p(1+p)^{T}\sum_{t=1}^{T}-(1+p)^{-1-t}t\left(-a_{t}+e_{t}\right)}{(b+g)\left(-1+(1+p)^{T}\right)}$$

Formula 4: Partial derivation of *ARR* according to *p*

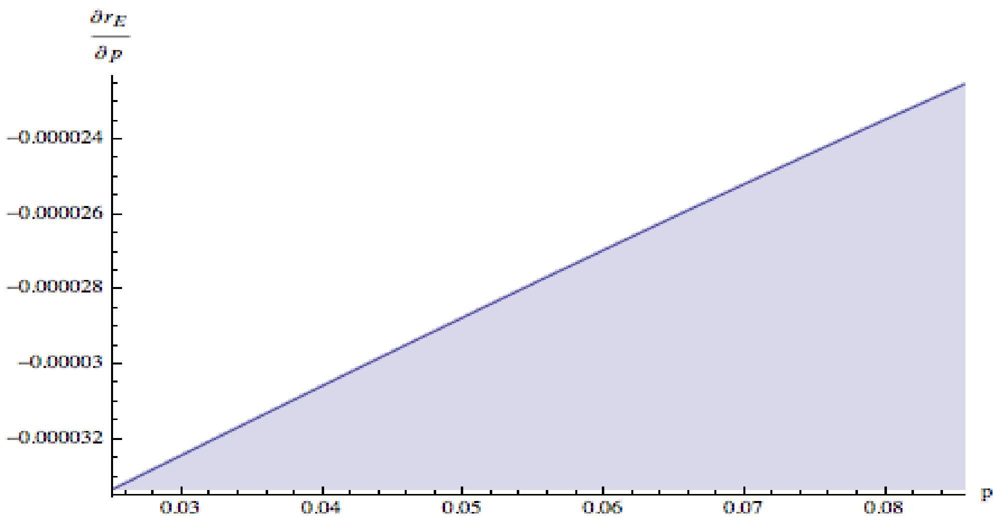

Image 6: Partial derivation of *ARR* according to *p*

The marginal effect of p on *ARR* is negative and decreases when *p* increases.

Partial derivation of ARR according to z

$$\frac{b}{b+g}$$

Formula 5: Partial derivation of *ARR* according to *z*

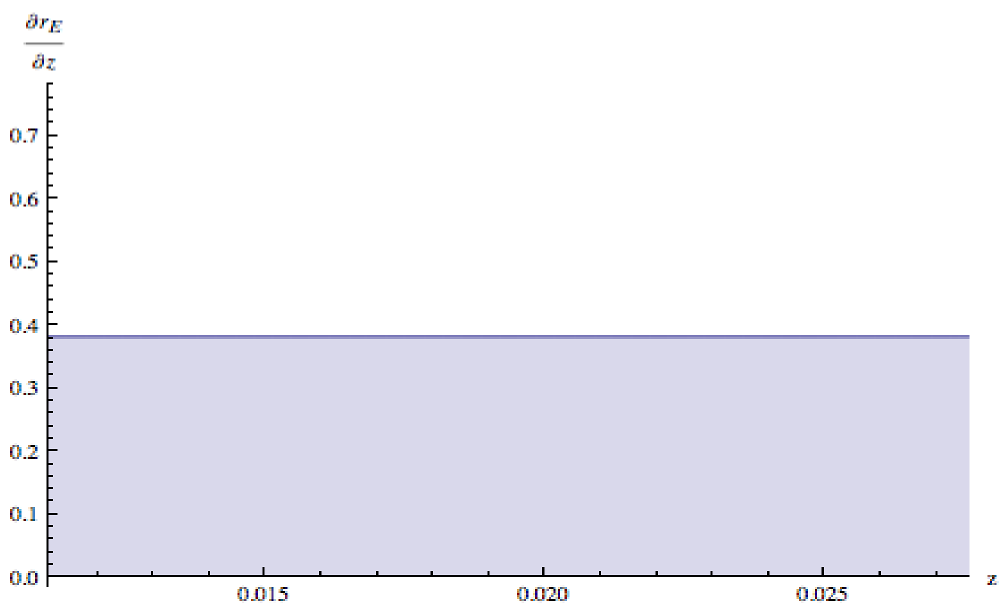

Image 7: Partial derivation of ARR according to z

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The marginal effect of *z* on *ARR* is quite surprising but very interesting for further implications. The marginal effect is essentially constant at *b / (b +g).* Therefor the marginal effect of *z* on *ARR* does not depend on *z* itself but only on *B* and *G* whereby *B* implies *z.*

Partial derivation of ARR according to T

$$-\frac{(1+p)^T \operatorname{Log}(1+p)\left(b\left(-1+(1+p)^T\right)z+p(1+p)^T\sum_{t=1}^{T}(1+p)^{-t}\left(-a_t+e_t\right)\right)}{(b+g)\left(-1+(1+p)^T\right)^2}+$$
$$\frac{p(1+p)^T\partial_T\sum_{t=1}^{T}(1+p)^{-t}\left(-a_t+e_t\right)+b(1+p)^T z\log(1+p)+p(1+p)^T\log(1+p)\sum_{t=1}^{T}(1+p)^{-t}\left(-a_t+e_t\right)}{(b+g)\left(-1+(1+p)^T\right)}$$

Formula 6: Partial derivation of *ARR* according to *T*

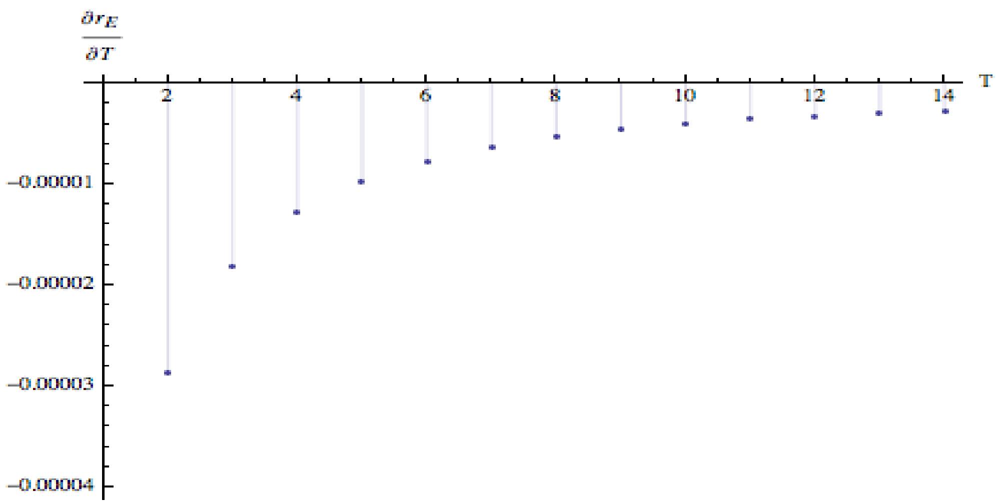

Image 8: Partial derivation of *ARR* according to *T*

The marginal effect of *T* on *ARR* is negative and decreases with an increasing *T*. Because the domain of *T* consists of discrete but not of steady values the graphical visualization deviates from the steady values.

Simulation Study

The goal of the simulation study is to simulate the independent input variables using empirically observed data on the basis of thus derived domains and to define the intensity and direction of the connections between input variables and *ARR* by forming non-parametric correlations. The following distributions were simulated for this:

p ~ N(0.0566; 0.0130)

z ~ N(0.0190; 0.0051)

b ~ U(10.000;450.000)

g ~ U(38.000;2.227.000)

T ~ U(1;14)

N = 10.000

Table 2: Distributions for the independent variables

While doing this the assumption was made that the simulated values are created independently from each other. This means that there is no correlation between the input factors. This may be an unrealistic assumption but it is there is no alternative due to the existing raw data.

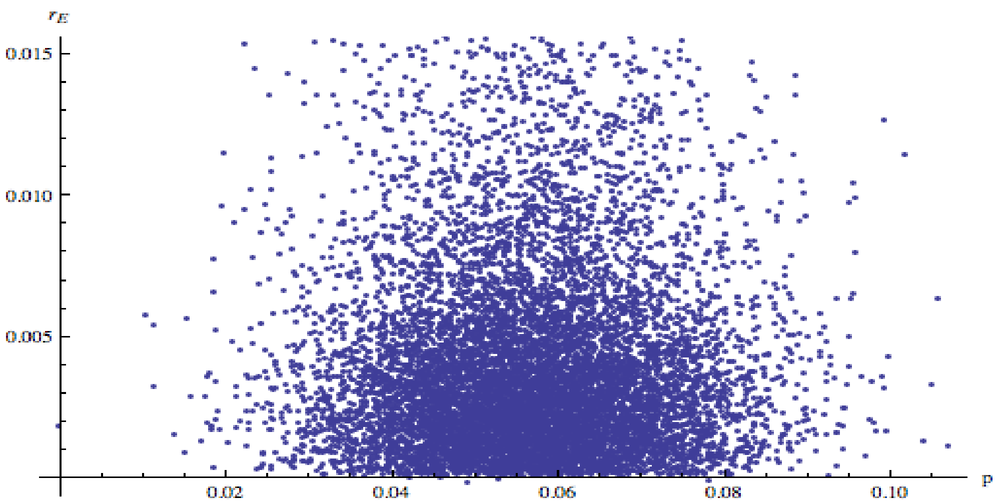

Image 9: Scatter diagram of the simulated values of *p* and *ARR*

No connection between *p* and *ARR* can be seen in the scatter plot. The point cloud is diffuse and unfocused.

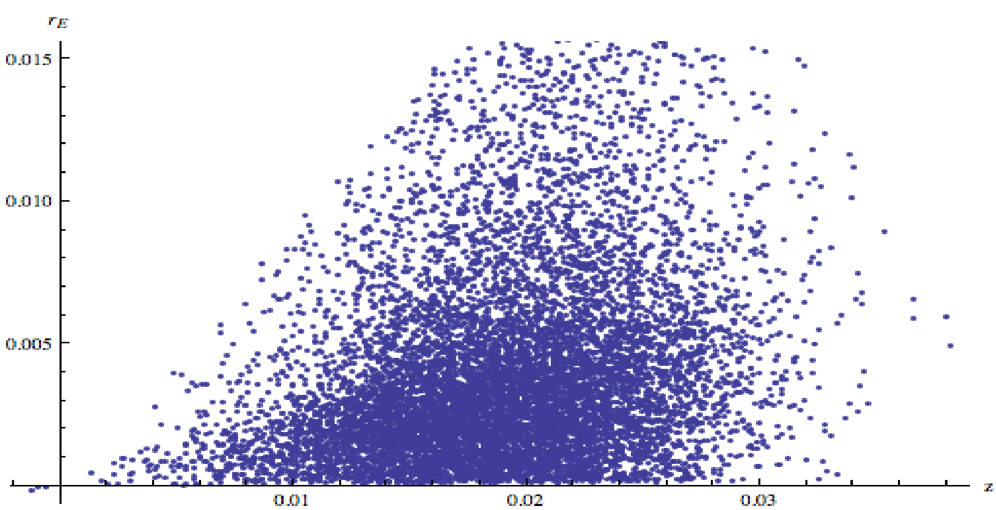

Image 10: Scatter diagram of the simulated values of *z* and *ARR*

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A positive connection between *z* and *ARR* is discernible in the scatter diagram. This corresponds with the constant, positive, marginal effect of the partial derivation.

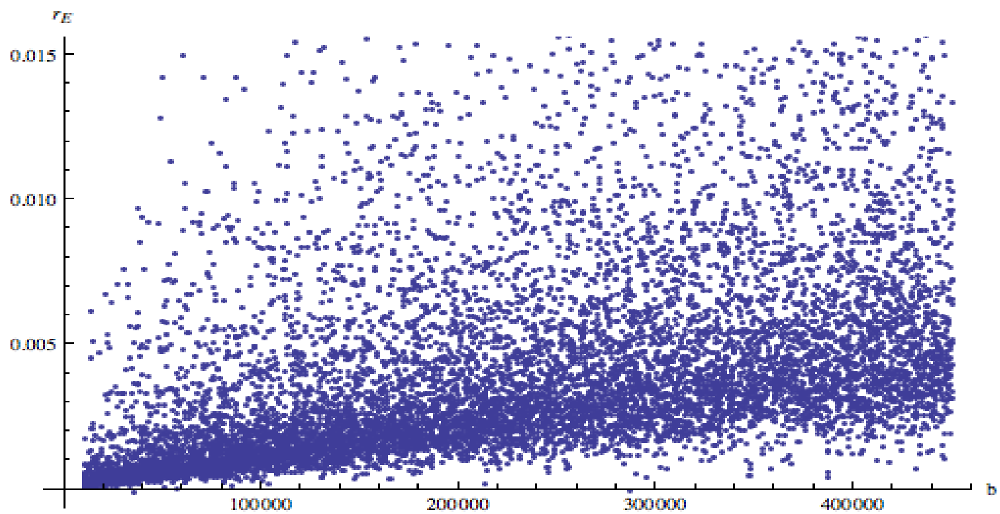

Image 11: Scatter diagram of the simulated vales of *B* and *ARR*

The scatter diagram also shows a positive connection between *B* and *ARR*. This also corresponds with the constant, positive and marginal effect of the partial derivation.

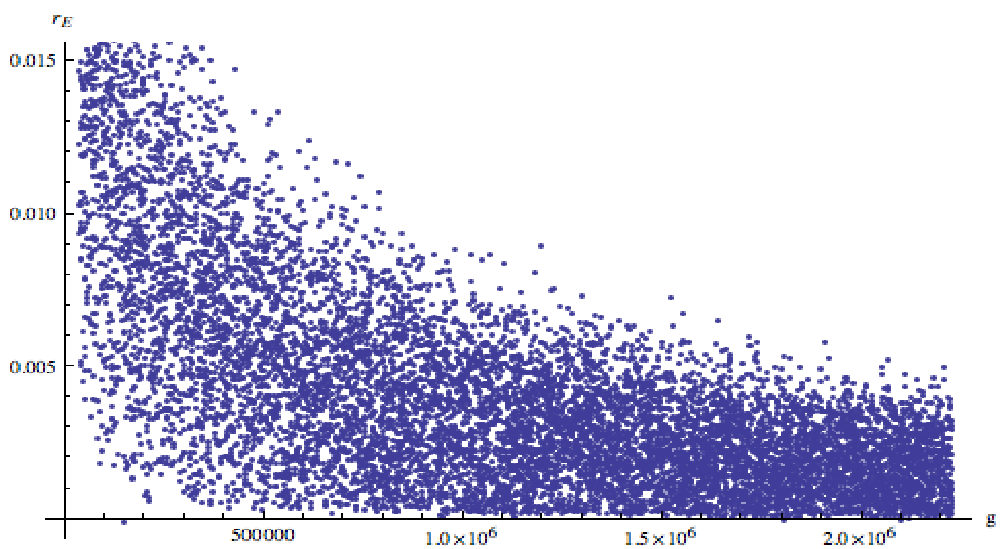

Image 12: Scatter diagram of the simulated values of *G* and *ARR*

A negative, non-linear connection between *G* and *ARR* can be seen in the scatter plot. This corresponds with the non-linear, negative marginal effect of the partial derivation.

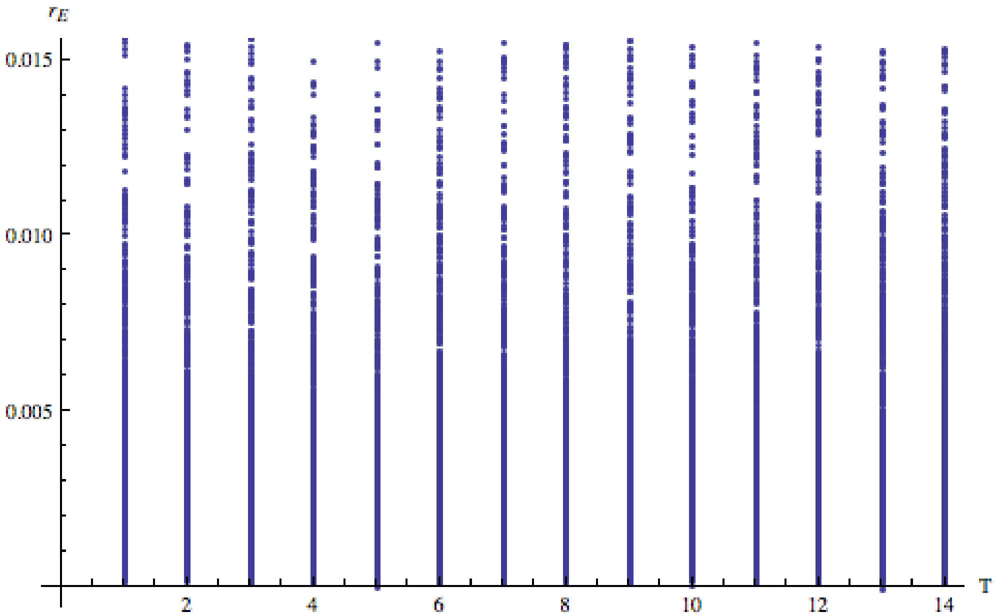

Image 13: Scatter diagram of the simulated values of *T* and *ARR*

No connection between *T* and *ARR* can be seen in the scatter diagram.

Finally non-parametric Spearman-correlations between the simulated values of the independent input variables and the respectively therefrom-resulting *ARR* were calculated.

Variable	*z*	*p*	*B*	*G*	*T*
ARR	0,011	0,304	0,617	-0,641	0,003

Table 3: Spearman-Correlations between *ARR* and independent input variables.

Here it can be seen clearly that *G*, *B* (which in turn determines *z*) and *p* are the determining input variables of the Asset Rate of Return. The effect of *T* is comparably low or zero in this particular simulation. It should however be considered that the simulated values are based on a model calculation.

The analysis above shows a simple linear positive effect of the combination *p* and *z* on *ARR*. It can be concluded that the ground has an independent value. This value directly influences the investment analysis of a real estate in the framework of this model and therefore must be included in the evaluation of a real estate analysis. The contemplation becomes more concrete with the analysis of the partial derivation. Here an almost steady marginal effect of *z* on *ARR* can be observed. Therefore the observed marginal effect can be defined as being constant and can itself be expressed through the partial derivation:

$b / (b+g)$.

Formula 7: marginal effect of *z* on *ARR*

It can be seen that the effect of *z* on *ARR* is not dependent on *z* itself but solely dependent on *B* and *G*. This means that in the end *z* is result of *B*. With this *z* can be described as being the factor that determines the amount of constant influence, which the ground value has on the Asset Rate of Return. This is also confirmed by the low correlation of *z* and *ARR* and by the high correlation of *B* and *ARR*.

In the course of the graphic depiction a positive linear effect of *p* in combination with *z* was found. The marginal effect of *p* on *ARR* is degressively negative. This means that an increasing *p* has a negative marginal effect on *ARR*, which decreases with an increasing *p* (runs degressively). The calculated Spearman correlation between *p* and *ARR* is 0,0304, which means that *ARR* is significantly determined by *p*. It can be seen that *p* functioning as weighted capital cost essentially determines the yield requirement. The higher *p* is the higher *ARR* must be. This results from the necessity to cover costs and to make profits. So there is a positive connection between *p* and *ARR* from the cost perspective. From the point of yields an increasing *p* influences the yield of a real estate investment negatively in the context of ADAM. This is shown with the marginal effect (partial derivation) of *p* on *ARR*. It can be concluded that an increasing calculation rate of *p* to cover costs and to make profits leads to a growth of the Asset Rate of Return. Because the calculation rate *p* presents the costs for borrowed capital and the yield requirements by the equity capital providers it implies a minimum for the Asset Rate of Return.

The analysis of the influence of *p* on *z* shows a simple linear connection at first. However after the analysis of the marginal effects on *ARR* over the partial derivation of *p* and *z* the conclusion must be different. The calculation rate *p* has a decreasing negative marginal effect on *ARR*. The ground yield rate *z* however has a constant marginal effect on *ARR*. The calculated non-parametric Spearman correlation with *ARR* is 0,011 for *z* and 0,304 for *p*. It can therefore be concluded that the influence of the ground yield rate *z* on the Asset Rate of Return is lower than the influence of the calculation rate *p* on *ARR* in the context of ADAM. However the influence of the ground value rate *z* (which in return is implied by ground value *B*) should not be neglected as can be seen above.

Conclusion

The methodology for real estate investment analysis (ADAM) (Schäfer, 2013) presented here is a further development of known methods for real estate evaluation and for real estate investment calculation. Real estates are goods with specific characteristics (heterogeneity, non replicable, fixed, etc.). The offer is therefore immobile and non-elastic which means that the demand of real estate markets must take over this function. Real estate markets (with direct real estate investments) are strongly segmented and locally influenced markets. Many submarkets exist due to the location, the type of real estate, the type of use etc. It can therefore be concluded that real estates and real estate markets differ strongly from the assumption of neoclassical microeconomics which assumes that all goods are homogenous and that markets are perfect. The German yield value calculation abstracts this special characteristic by including regional-specific parameters like the property interest rate or the ground value. It is therefore inevitable to observe the procedures for the value appraisal in order to create an investment model that suits the economic good real estate. The yield value calculation however does not consider the growing internationalization of real estate markets as well as the increasing connection of real estate to international capital markets. The DCF procedure embodies the capital market oriented approach but it does not consider the specific characteristics of the economic good real estate neither does it consider the interest payments or the income securities of the ground value. The mentioned contemplations give reason for the creation of a complex model for the evaluation of direct real estate investments such as the here presented model ADAM. Considering the different property laws for real estate within the EU such as the separation of ownership of ground and building (Handbuch Immobilienrecht in Europa, 2004) the ADAM can, as mentioned before, contribute to the aspired harmonization and increased transparency of the European real estate and capital markets according to the “European Directive on Markets in Financial Instruments 2004/39/EC”. Due to its general validity the model can be applied to individual practical cases and thereby provides results, which are relevant for common practice. Specifically the model can be applied to the ownership consolidation of real estate in the Czech Republic in 2014 (Hrncir, 2012), which was mentioned at the start of this paper. Due to the mathematical analysis the calculation rate *p* (which presents the capital market perspective) as well as the ground value *B* or rather the ground value yield *z* (which comes from the classical real estate evaluation or flow value calculation) were determined to be the essential influence factors for the evaluation of a direct real estate investment in the context of this paper. Therefore the derivation of the model from literature on evaluation, finance mathematical and investment theory has shown to be appropriate. Hence it can be concluded that the value of the ground and the therefrom-resulting partial yield should by all means be considered in the practice of evaluating real estate investment to enable a complete evaluation of real estate investments. In ADAM of course the tenancy agreement is the most determining influence factor of the yield of a real estate investment. However it can be seen that the value of the ground should not be neglected to be able to make a complete statement. Therefore the connection of capital market oriented and microeconomic-based evaluation or rather investment evaluation was successfully carried out. The special asset real estate can be evaluated completely investivly with this connection (microeconomic and capital market oriented). Due to the high complexity the possible correlations of the input variables were not considered in mathematical analysis that was conducted in this paper. For further research the correlation between the input variables could be added in the observation and analysis. In order to sharpen the analytic results the quantity and direction of the correlation of the input variables should be calculated. Since this would present an extremely large empirical research it could by all means be the object of additional scientific research.

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